

## Section 3D – Conclusions

### Vocabulary

**Population:** The collection of all people or objects to be studied.

**Sample:** Collecting data from a small subgroup of the population.

**Statistic:** A number calculated from sample data in order to understand the characteristics of the data.  
For example, a sample mean average, a sample standard deviation, or a sample percentage.

**Parameter:** A number that describes the characteristics of a population like a population mean or a population percentage. Can be calculated from an unbiased census, but is often just a guess about the population.

**Hypothesis Test:** A procedure for testing a claim about a population.

**Null Hypothesis ( $H_0$ ):** A statement about the population that involves equality. It is often a statement about “no change”, “no relationship” or “no effect”.

**Alternative Hypothesis ( $H_A$  or  $H_1$ ):** A statement about the population that does not involve equality. It is often a statement about a “significant difference”, “significant change”, “relationship” or “effect”.

**Population Claim:** What someone thinks is true about a population.

**Test Statistic:** A number calculated in order to determine if the sample data significantly disagrees with the null hypothesis. There are a variety of different test statistics depending on the type of data.

**Sampling Variability:** Also called “random chance”. The principle that random samples from the same population will usually be different and give very different statistics. The random samples will usually be different than the population parameter.

**P-value:** The probability of getting the sample data or more extreme because of sampling variability (by random chance) if the null hypothesis is true.

**Significance Level ( $\alpha$ ):** Also called the Alpha Level. This is the probability of making a type 1 error. The P-value is compared to this number to determine significance and sampling variability. If the P-value is lower than the significance level, then the sample data significantly disagrees with the null hypothesis and is unlikely to have happened because of sampling variability.

**Critical Value:** We compare a test statistic to this number to determine if the sample data significantly disagrees with the null hypothesis. If the absolute value of the test statistic is higher than the absolute value of the critical value, then the sample data significantly disagrees with the null hypothesis.

**Randomized Simulation:** A technique for visualizing sampling variability in a hypothesis test. The computer assumes the null hypothesis is true, and then generates random samples. If the sample data or test statistic falls in the tail, then the sample data significantly disagrees with the null hypothesis. This technique can also calculate the P-value and standard error without a formula.

**Conclusion:** A final statement in a hypothesis test that addresses the claim and evidence.

**Introduction:** So far we have learned many of the key ideas used in hypothesis testing. We learned how to write a null and alternative hypothesis and how to determine if the sample data significantly disagrees with the null hypothesis by comparing the test statistic to the critical value. We also learned how to compare the P-value and significance level in order to judge if the sample data occurred by random chance and whether to reject the null hypothesis or fail to reject.



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The last step in a hypothesis test is to write a formal conclusion. A conclusion is a statement about the claim the person made. Do we disagree with that claim (reject the claim) or do we agree with the claim (support the claim)? The conclusion is also a statement about evidence. Do we have evidence to back up our view about the population or not?

Conclusion: A final statement in a hypothesis test that addresses the claim and evidence.

*Note: "Reject  $H_0$ " or "Fail to reject  $H_0$ " is NOT a conclusion. This is a simple statement of what the P-value tells us about the null hypothesis. It does not address evidence and the claim.*

Writing conclusions can be difficult. Many statistics students struggle with the logic of the conclusion. Think of it this way.

Suppose we have a low P-value and  $H_0$  is the claim. A low P-value means we have evidence and will reject  $H_0$ . Since  $H_0$  is our claim, then our conclusion will be that we should "reject the claim". In other words, we have evidence that the random sample data significantly disagrees with the claim. In this case, our conclusion will be that "we have significant evidence to reject the claim".

Suppose we have a low P-value and  $H_A$  is the claim. Again, a low P-value means that we have evidence and we will reject  $H_0$ . Disagreeing with  $H_0$  is equivalent to agreeing with or supporting  $H_A$ . So our conclusion will be that "we have evidence to support the claim". In other words, random sample data significantly agrees with the claim.

Suppose we have a high P-value and  $H_0$  is the claim. A high P-value means that we do not have evidence and we will fail to reject  $H_0$ . Since  $H_0$  is our claim, then we should think of our conclusion as "failing to reject the claim". So our conclusion will be that "we do NOT have evidence to reject the claim". In this case, the random sample data is close to  $H_0$  and does not significantly disagree with the claim.

Suppose we have a high P-value and  $H_A$  is the claim. A high P-value is not evidence and means we will fail to reject  $H_0$ . You have to be able to reject  $H_0$  in order to support  $H_A$ . Since our random sample data does not significantly disagree with  $H_0$ , it also does not significantly agree with  $H_A$ . So our conclusion is that we do not have significant evidence to support the claim. The random sample data does not significantly agree with the claim.

Let's look at some steps for writing a formal conclusion to a hypothesis test.

#### Step 1: Address the Claim

**If the null hypothesis ( $H_0$ ) is the claim:** *There are two possibilities.*

- Yes, we have evidence to reject the claim  
OR
- No, we do not have evidence to reject the claim.

**If the alternative hypothesis ( $H_A$ ) is the claim:** *There are two possibilities.*

- Yes, we have evidence to support the claim  
OR
- No, we do not have evidence to support the claim.

#### Step 2: Address the evidence (Yes or No)

So how do we know if we have evidence or not?

We have already seen that a low P-value less than the significance level indicates significant evidence. Remember, scientists usually require a low P-value as evidence on their reports. If we have a high P-value greater than the significance level, then we do NOT have significant evidence.



Note: While P-value is the usual method for determining significance evidence, a test statistic in the tail also indicates significance evidence. For Z or T-test statistics, if the |test statistic| is larger than the |critical value| is considered significant. Remember a large test statistic in the tail will correspond to a small P-value. If the |test statistic| is smaller than the |critical value|, it is NOT considered significant.

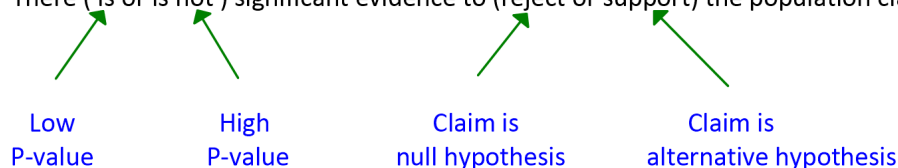
**Low P-value (less than significance level) : We have significant evidence.**

**High P-value (higher than significance level) : We do NOT have significant evidence.**

#### Step 3: Write the conclusion sentence

Remember a low P-value is considered significant statistical evidence but a high P-value is not evidence. When the claim is  $H_0$ , we will either be rejecting or not rejecting the claim. When the claim is  $H_A$ , we will either be supporting or not supporting the claim.

There ( is or is not ) significant evidence to (reject or support) the population claim.



#### Step 4: Explain the conclusion sentence

Our job as data scientists, statisticians and data analysts is to explain. People rarely understand the language and difficult ideas in statistics. The conclusion is a summary of the hypothesis test, but is rarely understood. It is always good to explain the conclusion in plain language.

There is significant evidence to reject the population claim. *(Random sample data significantly disagrees with the population claim and we have a low P-value as evidence.)*

There is significant evidence to support the population claim. *(Random sample data significantly agrees with the population claim and we have a low P-value as evidence.)*

There is not significant evidence to reject the population claim. *(Unfortunately, the hypothesis test was inconclusive. Random sample data does not significantly disagree with the population claim. We do not have any statistical evidence to disagree with the population claim. The claim might be correct.)*

There is not significant evidence to support the population claim. *(Unfortunately, the hypothesis test was inconclusive. Random sample data does not significantly agree with the population claim. We do not have any statistical evidence to agree with the population claim. The claim might be incorrect.)*

#### Example 1

A nursing magazine recently claimed that the population mean average amount of a particular medicine that is being given to patients is about 100 milligrams. Looking at a large random sample, we found a P-value of 0.0041 and a 5% significance level ( $\alpha = 0.05$ ) was used in the study. Assuming the data met all the assumptions, what would be the conclusion?

$H_0$ :  $\mu = 100$  milligrams (Claim)

$H_A$ :  $\mu \neq 100$  milligrams

*Step 1: Address the Claim*

The claim is  $H_0$ , so there are two possible conclusions.



- Yes, we have evidence to reject the claim  
OR
- No, we do not have evidence to reject the claim.

*Step 2: Address the Evidence*

The P-value = 0.0041 = 0.41% is less than the 5% significance level. So this is a low P-value close to zero. This means that sampling variability (random chance) is very unlikely and tells us the sample data significantly disagrees with the null hypothesis. We would reject the null hypothesis. Since the claim is the null hypothesis, we have evidence to reject the claim.

*Step 3: Writing the conclusion sentence:*

“There (*is or is not*) significant evidence to (*reject or support*) the claim.”

Notice this problem has a low P-value (“is evidence”) and claim is null (“reject”).

Formal Statistics Conclusion: “There is significant evidence to reject the claim that the mean average amount of this medicine given to patients is 100 mg.”

*Step 4: Explain the conclusion in plain language?*

Significant statistical evidence disagrees with the claim that the population mean average is 100 mg.

Example 2

An online article is currently estimating that more than 35% of people in the U.S. voted in the last election. Looking at a large random sample, we found a P-value of 0.267 and a 10% significance level ( $\alpha = 0.10$ ) was used in the study. Assuming the data met all the assumptions, what would be the conclusion?

$$H_0: p = 0.35$$

$$H_A: p > 0.35 \text{ (claim)}$$

*Step 1: Address the Claim*

The claim is  $H_A$ , so there are two possible conclusions.

- Yes, we have evidence to support the claim  
OR
- No, we do not have evidence to support the claim.

*Step 2: Address the Evidence*

The P-value = 0.267 = 26.7% is more than the 10% significance level. So this is a high P-value and is not close to zero. This data could have happened because of sampling variability (random chance) and tells us the sample data does not significantly disagree with the null hypothesis. We would fail to reject the null hypothesis. We do not have evidence to reject the null hypothesis. You have to reject the null hypothesis to be able to support the alternative hypothesis. So we do not have evidence to support the claim.

*Step 3: Writing the conclusion sentence:*

“There (*is or is not*) significant evidence to (*reject or support*) the claim.”

Notice this problem has a high P-value (“NOT evidence”) and claim is null (“support”).

Formal Statistics Conclusion: “There is not significant evidence to support the claim that more than 35% of people in the U.S. voted in the last election.”



*Step 4: Explain the conclusion in plain language?*

The hypothesis test was inconclusive. Random sample data does not significantly agree with the population claim. We do not have significant statistical evidence to agree with the population claim.

**Conclusion Summary (4 Possible Conclusions)**

If the claim is  $H_0$ , P-value is low (*Think “Yes Evidence Reject”*)

Conclusion Sentence: There is significant evidence to reject the claim.

(You are rejecting the null hypothesis and the null hypothesis is the claim.)

If the claim is  $H_0$ , P-value is high (*Think “No Evidence Reject”*)

Conclusion Sentence: There is not significant evidence to reject the claim.

(You do not have evidence to reject the null hypothesis and the null hypothesis is the claim.)

If the claim is  $H_A$ , P-value is low (*Think “Yes Evidence Support”*)

Conclusion Sentence: There is significant evidence to support the claim.

(*You are rejecting the null hypothesis which means you think the alternative hypothesis is correct and the alternative hypothesis is the claim.*)

If the claim is  $H_A$ , P-value is high (*Think “No Evidence Support”*)

Conclusion Sentence: There is not significant evidence to support the claim.

(*You do not have evidence to reject the null hypothesis which means you do not know if the alternative hypothesis is correct and the alternative hypothesis is the claim.*)

Important Notes:

- The claim being the null hypothesis does not in itself mean that you will reject the claim. It may be a “not reject” situation. You would only reject the claim if the claim is  $H_0$  and the P-value was low.
  - The claim being the alternative hypothesis does not in itself mean that you will support the claim. It may be a “not support” situation. You would only support the claim if the claim is  $H_A$  and the P-value was low.
  - Always address the “claim” in a conclusion. Never say that you support  $H_A$  or you reject  $H_0$ . That is not a conclusion.
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