

## Math 140 Final Review Sheet Answer Key

1. Vocabulary
  - a. Population
  - b. Bias
  - c. Random
  - d. Statistic
  - e. Sample
  - f. Simple Random Sample
  - g. Census
  - h. Cluster Sample
  - i. Stratified Sample
  - j. Convenience Sample
  - k. Voluntary Response Sample
  - l. Systematic Sample
  - m. Sampling Bias
  - n. Question Bias
  - o. Deliberate Bias
  - p. Response Bias
  - q. Non-response Bias
  - r. Observational Study
  - s. Single Blind
  - t. Placebo Effect
  - u. Double Blind
  - v. Control Group
  - w. Placebo
  - x. Experimental Design (Experiment)
  - y. Confounding Variables (Luring Variables)
  - z. Random Assignment
  - aa. Interquartile Range (IQR)

- ab. Median
- ac. Mean + Standard Deviation  
Mean – Standard Deviation
- ad. Mean
- ae. 1<sup>st</sup> quartile (Q1) and 3<sup>rd</sup> quartile (Q3)
- af. Standard Deviation
- ag. Standard Deviation
- ah. Median
- ai. 3<sup>rd</sup> quartile (Q3)
- aj. Mean
- ak. Interquartile Range (IQR)
- al. Minimum
- am. Mode
- an. Variance
- ao. Maximum
- ap. 1<sup>st</sup> quartile (Q1)
- aq. Range
- ar. Sampling Variability
- as. Bootstrap Distribution
- at. Confidence Interval
- au. Point Estimate
- av. Standard Error
- aw. 95% confidence
- ax. Sampling Distribution
- ay. 90% confidence
- az. Margin of Error
- ba. 99% confidence
- bb. Bootstrapping
- bc. P-value

- bd. Significance level (alpha level)
- be. Conclusion
- bf. Type 2 Error
- bg. Beta Level
- bh. Null Hypothesis
- bi. One-population mean T-test statistic
- bj. Alternative Hypothesis
- bk. One-population proportion Z-test statistic
- bl. Type 1 error
- bm. Critical Value
- bn. Two-population proportion Z-test statistic
- bo. Chi-square test statistic
- bp. Two-population mean T-test statistic
- bq. F-test statistic
- br. T-test statistic for correlation

2.

- a. Minimal sampling bias
- b. Significant sampling bias
- c. Significant sampling bias
- d. Minimal sampling bias
- e. Significant sampling bias

3.

We randomly assign the volunteers from the experiment into two or more groups. One group is the treatment group and the other is the control group. We attempt to make these groups as alike as we possibly can for all of the confounding variables. The only difference will be that one group received the treatment and the other group did not (received a placebo). Receiving the treatment or not is referred to as explanatory variable and the how people respond to the treatment is the response variable. If the groups are identical and the treatment group does significantly better than the control group, we have proven cause and effect. Medicine experiments should also be double blind with a placebo.

4.

- a. Categorical

b. Quantitative

c. Quantitative

d. Categorical

e. Quantitative

5.

a.  $2.4872\% = 2.5\%$

b.  $0.583\% = 0.6\%$

c.  $85.4226\% = 85.4\%$

6.

a. 0.063

b. 0.0000041

c. 0.527

7. Usually we want to put the categorical data into computer program. Can create a bar chart or a pie chart that would give the counts, proportion and percentages. In general, the proportion = amount / total and we can calculate the percentage by multiplying the proportion by 100%.

8.

a. Skewed left (negatively skewed)

b. Normally distributed (normal) (bell shaped)

c. Skewed Right (positively skewed)

9.

A normal data set has a bell shaped histogram. The center or average should be the mean. The spread should be the standard deviation. Always calculate the mean and standard deviation with a computer. Typical values are in between the mean – standard deviation and the mean + standard deviation.

Outliers are more than two standard deviations from the mean. The middle 68% of the data values are typical when the data is normal. The top 2.5% of the data values are unusually high (high outliers) and the bottom 2.5% are unusually low (low outliers).

10.

A skewed data set has a significantly longer tail to the right or left of the center. The center or average should be the median. The spread should be the IQR. Typical values are between Q1 and Q3. The middle 50% of data values are typical in skewed data. Outliers will be shown as little stars or circles on a box plot outside of the whiskers.

11.

To find the mean you add up all the numbers in your data set and divide by the sample size (n).

To find the variance, you subtract every number in the data set minus the mean. Square the differences and then add up the squares. Divide the sum of squares by the degrees of freedom (n – 1).

To find the standard deviation, take the square root of the variance.

Mean, variance and standard deviation when the data is normal (bell shaped).

12.

To find quartiles, put your data in order from smallest to largest. The center when the data is in order is the median. The center of the bottom half of the data is the 1<sup>st</sup> quartile and the center of the top half of the data is the 3<sup>rd</sup> quartile. IQR is calculated by subtracting Q3 – Q1. The median is usually a very accurate average or center for almost any data set. But quartiles and IQR are used primarily for non-normal or skewed data.

13.

Letter	Statistic or Parameter?	Standard Deviation, Mean, Proportion, or Frequency?
$\hat{p}$	Statistic	Proportion
N	Parameter	Frequency
$\mu$	Parameter	Mean
s	Statistic	Standard Deviation
n	Statistic	Frequency
$\sigma$	Parameter	Standard Deviation
$\pi$	Parameter	Proportion
$\bar{x}$	Statistic	Mean

14.

Random Sample

Individuals within the sample are independent

Sample size at least 30 or normal

15.

Random Samples

Individuals within the sample and between the samples are independent

Both samples have a sample size at least 30 or normal

16.

Random ordered pair sample

Individuals within the ordered pair sample are independent

Sample differences have a size at least 30 or normal

17.

Random Sample

Individuals within the sample are independent

At least 10 successes

At least 10 failures

18.

Random Samples

Individuals within the sample and between the samples are independent

Both samples have at least 10 successes

Both samples have at least 10 failures

19.

We take many random samples from a population, calculate many sample statistics, and then put all the statistics on the same graph. This is a sampling distribution. The sample statistics (mean and proportion) were usually very different than each other and different than the population parameter. That is the principle of sampling variability. The center of a sampling distribution is close to the population parameter. The standard deviation of the sampling distribution is the standard error.

It is important for a sampling distribution to be normal since the mean of the sampling distribution and the standard deviation of the sampling distribution would only be accurate if it is normal. Also Z-score and T-score critical values come from normal curves.

20.

Central Limit Theorem: A sampling distribution for sample means or sample proportions will be normally distributed if the sample size is large enough.

Large Enough?

Sample Means: Sample size at least 30

Sample Proportions: At least 10 successes and at least 10 failures

21.

A confidence interval is two numbers that we think the population parameter is in between.

Multiply the Z or T-score critical value by the standard error to get the margin of error.

Take the sample statistic and add and subtract the margin of error to get the upper and lower limits of the confidence interval.

It is important for the data to meet the assumptions so that we do estimate population parameters from bad biased data. Also the critical values and standard error will not be accurate.

22.

Bootstrapping takes lots of random samples with replacement from one original real random sample. It creates a bootstrap distribution from the bootstrap statistics. The middle 95% of the bootstrap distribution will represent the confidence interval. The two numbers at the bottom of the distribution are the upper and lower limits of your confidence interval.

23.

If the two-population confidence interval is (+,+), then population1 is significantly larger than population 2. It also tells us that it is between those two numbers larger.

If the two-population confidence interval is (negative,negative), then population1 is significantly smaller than population 2. It also tells us that it is between those two numbers smaller.

If the two-population confidence interval is (negative, positive), then there is no significant difference between the two population.

24.

Increased sample size will decrease the standard error and the margin of error, which will give a narrower confidence interval.

Decreased sample size will increase the standard error and the margin of error, which will give a wider confidence interval.

25.

If the confidence level increases, critical value gets larger and the margin of error will increase. This will make a wide confidence interval.

If the confidence level decreases, critical value gets smaller and the margin of error will decrease. This will make a narrow confidence interval.

26.

Z-scores and T-scores both count the number of standard errors. If the sample size is small, T-scores will be significantly larger than the Z-scores. If the sample is large, T-scores and Z-scores are about the same.

Use Z-scores for one and two-population proportions.

Use a T-score for one and two-population means and the correlation test.

27.

Margin of Error tells us how far off a sample statistic could be from the population parameter. To get the confidence interval, add and subtract the sample statistic and margin of error.

95% One-population mean confidence interval: (\$56.75 , \$69.33)

Sentence: We are 95% confident that the population mean is in between \$56.75 and \$69.33.

90% Two-population proportion confidence interval: (-0.084 , -0.055)

We are 90% confident that the population proportion for group 1 is between 0.055 and 0.084 lower than the population proportion for group 2.

28.

The statement the article or person believes is true is the "claim". We write down the claim and the opposite of the claim. The statement with = is the null hypothesis ( $H_0$ ) and the statement that does not have equality is the alternative hypothesis ( $H_a$ ).

$H_a$ : > (right tailed test)

$H_a$ : < (left tailed test)

$H_a$ : not equal (two-tailed)

29.

a) One-population proportion

Assumptions:

Random Sample

Individuals within the sample are independent

At least 10 successes

At least 10 failures

$H_0$ :  $\pi = 0.25$

$H_a$ :  $\pi > 0.25$

Z-test statistic



b) One population mean

Random Sample

Individuals within the sample are independent

Sample size at least 30 or normal

Ho:  $\mu = 98.6$

Ha:  $\mu < 98.6$

T-test statistic

c) Two-population proportion

Assumptions:

Random Samples

Individuals within and between the samples are independent

Both samples have At least 10 successes

Both samples have At least 10 failures

Ho:  $\pi_1 = \pi_2$

Ha:  $\pi_1 \neq \pi_2$

Z-test statistic

d) Two-population mean (separate groups)

Random Samples

Individuals within and between the sample are independent

Both Sample size at least 30 or normal

Ho:  $\mu_1 = \mu_2$

Ha:  $\mu_1 \neq \mu_2$

T-test statistic

e) Two population mean (matched pairs)

Assumptions

Random ordered pair sample

Individuals within the ordered pair sample are independent

Sample differences have a size at least 30 or normal

Ho:  $\mu_d = 0$

Ha:  $\mu d \neq 0$

T-test statistic

f) Goodness of Fit

Random Samples

Individuals within and between the samples are independent

Expected counts at least 5.

Ho:  $\pi_1 = \pi_2 = \pi_3 = \pi_4$

Ha: at least one  $\neq$

Chi-square test statistic

g) ANOVA

Random Samples

Individuals within and between the samples are independent

Sample sizes at least 30 or normal

Sample standard deviations should be close.

Ho:  $\mu_1 = \mu_2 = \mu_3 = \mu_4$

Ha: at least one  $\neq$

F-test statistic

h) Categorical Association Test

Random Samples

Individuals within and between the samples are independent

Expected counts at least 5.

Ho: categorical variables are not related

Ha: categorical variables are related

Chi-square test statistic

i) Correlation

Assumptions

Random Quantitative ordered pairs samples

Individuals within the samples are independent

Sample size at least 30

Scatterplot shows some linear trend and no influential outliers

Residual plot evenly spread out

Histogram of the residuals normal and centered close to zero.

Ho: Population slope = 0

Ha: Population slope  $\neq$  0

T-test statistic

30.

Categorical / Categorical is Categorical Association test

Categorical / Quantitative is the ANOVA

Quantitative / Quantitative is Correlation

31.

If the test statistic falls in the tail determined by the critical value then the sample data significantly disagrees with the null hypothesis.

32.

<i>Type</i>	<i>Test Statistic</i>	<i>Critical Value</i>	<i>Does the Test Statistic fall in a tail determined by the critical value or not?</i>	<i>Does the sample data significantly disagree with <math>H_0</math> or not?</i>	
<i>Right-tailed</i>	$\chi^2 = +6.327$	$+9.833$	Not in Tail	Not significantly disagree	Are the observed counts significantly different than the expected counts? No. Not significantly different
<i>Two-tailed</i>	$Z = +2.46$	$\pm 1.96$	In tail	Significantly disagrees	Is the sample proportion for group 1 significantly higher than group 2? Yes. Significantly higher
<i>Left-tailed</i>	$T = -1.377$	$-2.643$	Not in Tail	Not significantly disagree	Is the sample mean for group 1 significantly lower than group 2? No. Not significantly lower.
<i>Right-tailed</i>	$F = +4.612$	$+3.182$	In Tail	Significantly disagree	Is the variance between significantly higher than the variance within? Yes. Significantly higher

33. If the null hypothesis is true, the P-value is the probability of getting the sample data or more extreme by sampling variability. If the P-value is less than or equal to the significance level, the sample data significantly disagrees with the null hypothesis, we will reject the null hypothesis, and the sample data was unlikely to occur by sampling variability. If the P-value is higher than the significance level, then the sample data does not significantly disagree with the null hypothesis, the sample data could have occurred by sampling variability, and we will fail to reject the null hypothesis.

34.

<i>P-value</i>	<i>P-value %</i>	<i>Significance Level (<math>\alpha</math>)</i>	<i>Significance Level %</i>	<i>Is the P-value lower or higher than the sig level?</i>	<i>If the null hypothesis was true, could this sample data occur by sampling variability or is it unlikely?</i>	<i>Reject <math>H_0</math> or Fail to reject <math>H_0</math>?</i>	<i>Significant Evidence or not?</i>
0.00248	0.248%	0.01	1%	Lower	Unlikely	Reject $H_0$	Sig Evidence
$3.4 \times 10^{-5}$	0.0034%	0.10	10%	Lower	Unlikely	Reject $H_0$	Sig Evidence
0.447	44.7%	0.05	5%	Higher	Could be	Fail to reject $H_0$	Not Sig

35.

If the null hypothesis is the claim, then the conclusion will either be

“There is significant evidence to reject the claim.”

OR

“There is NOT significant evidence to reject the claim.”

If the alternative hypothesis is the claim, then the conclusion will either be

“There is significant evidence to support the claim.”

OR

“There is NOT significant evidence to support the claim.”

A low P-value indicates that there is significant evidence.

A high P-value indicates that there is NOT significant evidence.

36.

a. There is NOT significant evidence to reject the claim.

(The claim could be correct, but we do not have evidence.)

b. There is significant evidence to reject the claim.

(We have evidence that the claim may be wrong.)

c. There is NOT significant evidence to support the claim.

(The claim could be wrong, but we do not have evidence.)

d. There is significant evidence to support the claim.

(We have evidence that the claim may be correct.)

37.

In a randomized simulation or a randomization technique, the computer creates thousands of simulated samples under the premise that the null hypothesis is true. It calculates the statistic from each of the simulated samples and puts them on the same graph. So a simulated sampling distribution is created under the premise that the null hypothesis is true. The center of the distribution will be very close to the null hypothesis parameter. The standard deviation of the sampling distribution gives us an estimate of the standard error. In a sense we have a graph describing sampling variability (random chance) if the null hypothesis is true.

To find the critical value or tail corresponding to the significance level ( $\alpha$ ), click the appropriate tail (left tail, right tail or two tail) on the simulation and put in the significance level in the top probability box. If it is a two tailed test put  $\frac{\alpha}{2}$  in each tail. The bottom box will be the start of the tail or tails or the critical value or values.

To find the P-value, put the original sample statistic or the test statistic in the bottom box of the tail. The top box will be the P-value. If it is a two tailed test, you will need to add the top probability boxes.

The assumptions for randomized simulation are random samples and individuals independent.

38.

A type 1 error is believing that the alternative hypothesis is correct when in fact it is not. Sample data indicates that we should support the alternative hypothesis, but this is data that was biased and not reflective of the population.

The probability of type 1 error is the significance level (alpha level). To decrease the chances of a type 1 error, decrease the significance level.

A type 2 error is believing that the null hypothesis is correct when in fact it is not. Sample data indicates that we should fail to reject the null hypothesis, but this is data that was biased and not reflective of the population.

The probability of type 2 error is the beta level. To decrease the chances of a type 2 error, increase the sample size.