Hypothesis Test Notes: Type 1 and Type 2 Errors

Sampling Variability can sometimes really mess up a hypothesis test. When that happens, there can be severe consequences. Type 1 and Type 2 errors occur when the sample data is not reflective of the population and gives us a wrong view about the population.

Type 1 Error: Think the alternative hypothesis H_A is correct when it is not.

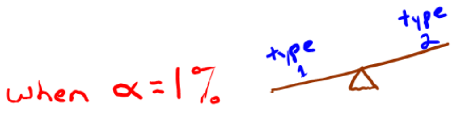
- A biased random sample gives low P-value that is not reflective of the population. The person analyzing the data thinks that the null hypothesis is wrong and the alternative is correct when in reality it is not. The data analyst rejects H_0 and supports H_A when in actuality, the null hypothesis H_0 is correct and the alternative hypothesis H_A is wrong.

Type 2 Error: Think the null hypothesis H_0 is correct when it is not.

- Biased random sample gives high P-value that is not reflective of the population. The person analyzing the data then fails to reject H_0 and thinks H_0 might be correct), when in actuality, the null hypothesis H_0 is wrong and the alternative hypothesis H_A is correct.

How to stop a Type 1 Error? *Lower the significance level!!*

- The significance level (alpha level) is the probability of type 1 error. So to limit the chances of a type 1 error, simply lower the significance level from 5% to 1%.
- Important note: Type 1 and Type 2 are on a see-saw. As one goes up the other goes down. If you decrease the significance level from 5% to 1%, the probability of type 1 error (alpha level) goes down, but the probability of type 2 error (beta level) will now increase.



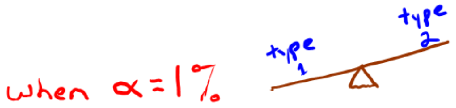
How to stop a Type 2 Error? *Increase the sample size (collect more sample data)* *or Increase the significance level*

- The probability of type 2 error is called a beta level. To decrease the beta level, most statisticians increase the sample size. More data results in a more powerful test and a lower probability of type 2 error (decreased beta level).
- If you cannot collect more data, another option is to increase the sample size. Remember Type 1 and Type 2 are on a see-saw. As one goes up the other goes down. If you increase the significance level from 5% to 10%, the probability of type 1 error will increase, but the probability of type 2 error (beta level) will now decrease.

Significance levels and type 1 and type 2 errors

- 5% significance level (95% confidence level) is a good balance between type 1 and type 2 errors. Both are relatively low. *(This is why most hypothesis tests use a 5% significance level and most confidence intervals use a 95% confidence level.)*

- 1% significance level (99% confidence level) will have a lower probability of type 1 error but a higher probability of type 2 error.



- 10% significance level (90% confidence level) will have a higher probability of type 1 error but a lower probability of type 2 error.

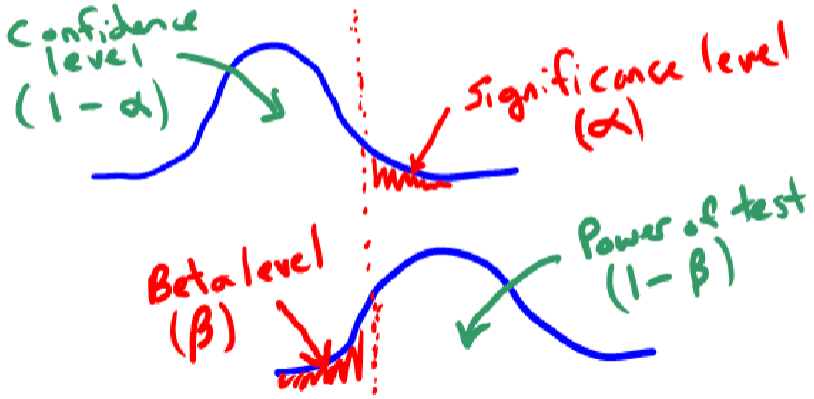
Summary of Type 1 Error

- Type 1 error is believing that the alternative hypothesis (H_A) is correct when it is not.
- Type 1 error happens when biased sample data gives a low p-value that does not reflect the population.
- The probability of type 1 error is the significance level (alpha level α).
- To limit the chances of type 1 error, decrease the significance level *(decrease the alpha level)*

Summary of Type 2 Error

- Type 2 error is believing that the null hypothesis (H_0) is correct when it is not.
- Type 2 error happens when biased sample data gives a high p-value that does not reflect the population.
- The probability of type 2 error is called the beta level β.
- To limit the chances of type 2 error, either increase the sample size or increase the significance level (increase the alpha level).

Diagram of Alpha levels, Beta levels, confidence levels, and power.



Note: When exploring type 1 and type 2 errors, it is important to write down the null and alternative hypothesis and the consequences of believing the null is true and the consequences of believing the alternative is true.

Example

A pharmaceutical company wants to sell a new medicine in the U.S. To get approval they need to convince the FDA that the medicine is safe and has few side effects. If serious side effects happen in 3% or more of the people taking the medicine, then the FDA may not approve sale of the medicine in the U.S. If serious side effects happen in less than 3% of people taking the medicine, then the FDA may approve sale of the medicine in the U.S.

What is the null and alternative hypothesis?

Ho: $p \geq 3\%$ (FDA does not allow medicine to be sold in U.S.)

Ha: $p < 3\%$ (FDA does allow medicine to be sold in U.S.)

Describe the consequences of a type 1 error and what we could do to limit the probability of a type 1 error.

Because of some biased sample data, we got a low P-value and think that the alternative hypothesis is correct when it is not. (In reality, the null hypothesis is correct.) That would mean that the FDA approved sale of the medicine by mistake. The medicine causes serious side effects in alot of people. People could die or become very sick. They may sue the pharmaceutical company or the FDA.

To make sure this doesn't happen, lower the significance level to 1% (or even lower).

Describe the consequences of a type 2 error and what we could do to limit the probability of a type 2 error.

Because of some biased sample data, we got a high P-value and failed to reject the null hypothesis when the null hypothesis is really wrong. (In reality the alternative hypothesis is correct.) So we think that the null hypothesis is correct when it is not. That would mean that the FDA blocked the sale of a good medicine that rarely causes any side effects. Patients will be deprived of a good medicine and the company will lose a lot of money in potential profits.

To make sure this doesn't happen, increase the sample size. We should NOT increase the significance level in this case because if we make a type 1 error, it would have terrible consequences.)