

## Introduction to Confidence Intervals

How to figure out a population parameter?

- Unbiased Census → Can get the population parameter.
- Sampling Distributions (lots of random samples) → Center of distribution is a pretty good estimate of Population Parameter

In the real world, we usually have only one random sample

- The random sample statistic could be very off from the population parameter.
- How far off?
- Estimate Margin of Error (How far off the sample statistic could be from the population parameter.)

If we can estimate the margin of error, we can create something called a confidence interval.

**Confidence Interval**: Two values we think the population parameter is in between.

How to Calculate a Confidence interval:

**Sample Statistic  $\pm$  Margin of Error**

*(Note: This formula only works if the sampling distribution is normal.)*

**Confidence Level : 90%, 95% and 99% (most common is 95%)**

**Define “95% confident”**: 95% of confidence intervals created contain the population parameter and 5% of them don't contain the population parameter.

**Define “90% confident”**: 90% of confidence intervals created contain the population parameter and 10% of them don't contain the population parameter.

**Define “99% confident”**: 99% of confidence intervals created contain the population parameter and 1% of them don't contain the population parameter.

### **Example 1**

**IQ tests: Sample mean of 99 and a margin of error of 26.**

*(Used a 95% confidence Level)*

**Create Confidence interval ( $99 \pm 26$ )**

Sample mean – margin of error < population mean < sample mean + margin of error

$99 - 26 < \text{population mean} < 99 + 26$

$73 < \text{population mean} < 125$

**Sentence to explain this confidence interval?**

**“We are 95% confident that the population mean average IQ score is between 73 and 125.”**

**Write Confidence intervals in three ways**

- 1. *Sample Value  $\pm$  margin of error***  
**99 ( $\pm 26$  error)**
- 2. *Inequality notation:  $73 < \mu < 125$***
- 3. *Interval notation: ( 73 , 125 )***

## Example 2

Sample percentage was 0.365 (36.5%)

Margin of Error = 0.049 (4.9%)

*(Used a 90% confidence level)*

Confidence interval: 0.365 + or – 0.049

36.5% (+or- 4.9% error)

$0.365 - 0.049 < \text{population percentage} < 0.365 + 0.049$

$0.316 < P < 0.414$

( 0.316 , 0.414 )

Sentence to explain interval? We 90% confident that the population percentage is in between 31.6% and 41.4%.

### Note:

Programs and scientific reports sometimes give the confidence interval without the Margin of Error. Suppose we know the confidence interval. Can we figure out the sample statistic and margin of error? Sure

The sample statistic will be at the middle of the interval. The margin of error is the distance from the middle. Here are two formulas commonly seen in stat books.

Sample Statistic = ( upper limit + lower limit ) / 2

Margin of Error = ( upper limit - lower limit ) / 2

### **Example 3**

**Suppose we have a 99% confidence interval estimate of the population mean weight (in kilograms)**

**(51.7 kg , 63.4 kg)**

**Sentence? We are 99% confident that the population mean weight is between 51.7 kg and 63.4 kg.**

**What sample mean and margin of error were used to make this confidence interval?**

**Sample Statistic (sample mean) =  $(63.4 + 51.7) / 2 = 115.1 / 2 = 57.55$  kg**

**Margin of Error =  $(63.4 - 51.7) / 2 = 11.7 / 2 = 5.85$  kg**