

Hypothesis Test Notes

P-value and Significance Levels

Hypothesis Test: Using Random Sample data to decide whether a population claim seems reasonable or if it looks very wrong.

Problem: Sampling Variability!!

Random samples are usually different and random samples are usually very different than the population value. There are two possibilities and we will need to decide which one we think is correct.

Option 1: Is our random sample data different than the population value because all random samples are different (random chance)? In which case the population value might be correct.

OR

Option 2: Is our random sample data different than the population value because the population value is wrong.

Important Note:

Think of random chance (sampling variability) as a confounding variable. In order to show that the population value in H_0 is wrong (option 2), we have to rule out random chance (sampling variability). In other words we have to make sure option 1 is not correct (or at least highly unlikely), to be able to say that population value is probably wrong (option 2).

P-value to the rescue!!

P-value can help us decide between the two options.

Definition

P-value : The probability of getting the sample data or more extreme by random chance if the null hypothesis is true.

Probability & Logic principle: If the probability of an event happening is very low, but the event keeps happening, then we should look for a different explanation. Our assumption about how that event works might be wrong.

Assumption: Suppose the population value is correct (in null hypothesis H_0). The P-value calculates the probability of getting the sample data by random chance based on that assumption.

Low P-value

If the P-value is very low, then the sample data probably did not happen by random chance. This means option 1 is very unlikely. So probably option 2 is true and our assumption about population value being correct in the null hypothesis is probably wrong. When that happens we say we “Reject the Null Hypothesis”.

High P-value

If the P-value is high, then the sample data could have happened by random chance. The sample data is probably different because of sampling variability (all random samples are different). Option 1 could be true, meaning we cannot rule out random chance (sampling variability) as a confounding variable. So we will not be able to tell if the population value is wrong. Since it is likely that option 1 occurred, the population value in the null hypothesis might be correct.

Since we can't tell if the population value is right or wrong, we say we "Fail to Reject the Null Hypothesis".

Important Notes:

- *Failing to reject H_0 does not mean that H_0 is true!! It means we cannot tell if the population value is right or wrong. Sampling Variability has struck again.*
- *A low P-value occurs when the sample value significantly disagrees with the population value in the null hypothesis. In other words a low P-value corresponds with a large test statistic. Both mean that the population value is probably wrong.*
- *A high P-value occurs when the sample value is pretty close to the population value in the null hypothesis. In other words a high P-value corresponds with a small test statistic. Both mean that population value might be correct.*

Significance Levels

Sometimes a P-value might be border line. Remember we want the P-value to be low to insure that the sample data was unlikely to occur by random chance. But how low do we need it?

Significance Levels (also called alpha levels)

α (*Greek Letter Alpha*)

Significance levels (α) are a number we can compare the P-value too. We will also see later they are also associated with avoiding certain types of errors in statistics.

Remember confidence levels? Significance levels (α) are the opposite of confidence levels ($1-\alpha$). If you want to be 95% confident for example the significance level would be $100\%-95\% = 5\%$. This is the most common significance level used.

Common Confidence Levels and Significance Levels.

Confidence Level	Significance Level
90% (0.90)	10% (0.10)
95% (0.95)	5% (0.05)
99% (0.99)	1% (0.01)

So before you do your hypothesis test you should choose which significance level you want to use. If you are unsure, use 5% as this is the most common.

Summarize:

If the P-value \leq significance level, Reject the null hypothesis.

If the P-value $>$ significance level, Fail to reject the null hypothesis

Take a look at top half of the “P-value Diagram”.

(Can be found on the website on the hypothesis test page.)

Summary:

Low P-value

- Sample data significantly different than the population value
- Sample data probably did not happen by random chance (sampling variability)
- Reject H_0

High P-value

- Sample data close to the population value (not significantly different)
- Sample data could of happened by random chance (sampling variability)
- Fail to Reject H_0 (Does not mean H_0 is true)

Example 1 (Write the null and alternative hypothesis and interpret what we can learn from the given p-value. Assume the problem meets assumptions and use a 5% significance level)

An article on line said that the average typing speed for all adults is about 40 (words per minute). We took a large random sample in order to test this claim. Our sample mean was 38 (words per minute) and our P-value was 0.216

$$H_0 : \mu = 40 \text{ (claim)}$$

$$H_A : \mu \neq 40$$

P-value $0.216 > 0.05$ sig level

Fail to reject H_0

(Average typing speed might be 40 might be correct but we are not sure)

Sample mean was not significantly different than population value of 40.

The random sample mean of 38 (wpm) could have happened by random chance.

Example 2 (Write the null and alternative hypothesis and interpret what we can learn from the given p-value. Assume the problem meets assumptions and use a 1% significance level)

A pharmaceutical company is developing a new medicine to help people with diabetes. They want to see if the medicine will help at least 50% of people that take it. They took a random sample of people taking the medicine. They got a P-value of 8.74×10^{-4} .

$$H_0 : p \geq 50\% \text{ (claim)}$$

$$H_A : p < 50\%$$

Notice the P-value is in scientific notation.

$$8.74 \times 10^{-4} = 0.000874$$

P-value $0.000874 < \text{sig level } 0.01$

Reject H_0

It is probably not true that the medicine helps at least 50% of people that take it.

Data supports that it is less than 50%

Sample data was significantly different than population percent (50%).

The random sample data is very unlikely to occur by random chance.