`Confidence Interval & Sampling Distribution Answer Keys

Conf int act 1

1. Answers may vary. Many students are surprised to find their random sample mean is very different than the population value of 134.338 cents. The random sample means vary between 110 cents and 160 cents (see picture). For example if a student got a sample mean of 110 cents, then their random sample mean is 24.338 cents too low.

2. Most of the sample means on the board (see picture) are vastly different than the population mean of 134.338 cents. This happens because there is a lot of variability in random samples. Even though random sampling is one of the best ways to collect data, this does not guarantee that the random sample value will be close to the population value. Random samples are always different and usually very different than the population value.

3. If all we knew is one random sample, it would be practically impossible to predict the population value with any accuracy. Most of the random samples means were very far off. If all we knew is the random sample mean, it would be almost impossible to know the population value with any certainty.

4. The shape of sampling distribution is relatively bell shaped (nearly normal). Yes. The center of the sampling distribution is much closer to the population value of 134.338 cents and would be a much better approximation of the population value than one single random sample mean like 110 cents.

5. 95% of 81 = 0.95 x 81 = 76.95 (approx. 77)

So we want the middle 77 dots and must take off a total of 4 dots (81-77). So taking two from each end gives us the approximate values of 156 and 111. The distance between them is 45. The empirical rule states that the middle 95% of a bell shaped curve corresponds to 2 standard deviations above and 2 standard deviations below the center. So that is a total of 4 standard deviations. Take the distance of 45 and divide by 4. This gives an approximate standard deviation of 11.25. The standard deviation of a sampling distribution is called the “standard error”. So the standard error is approximately 11.25. (Answers may vary.)

Conf int act 2

1. Sampling variability is on display. Notice that most of the random sample means are vastly different than the population mean. A point estimate is when people take a random sample value and tell people it is the population value. This is horrible. Most of these random sample means are not even close to the correct population value.

2. Answers will vary. That is the point. Random samples are always different and always different than the population value. Students can take as many samples as they want. For example they may take 2000 random samples. As the number of samples in the distribution increases, it is becoming more and more bell shaped (central limit theorem in action). The population is skewed, but the sampling distribution of sample means still comes out very bell shaped.

3. Answers will vary. The center of the distribution gets more and more accurate as the number of random samples increase. This is a general principle in stats. The more random data you have, the less the error. The center of the distribution for example might be 134.6 cents. This is extremely close to the population. So to get a really good estimate of the population all you need is thousands of random samples. (This is usually not possible. In real life, you may have only 1 random sample.)
4. The standard deviation of a sampling distribution is called the standard error. Answers will vary but most standard errors will be around 10. One example was 10.104.

Note the difference between the standard deviation (of 1 random data set, vary, some came out to 55 to 66) and the standard error (standard deviation of thousands of sample values came out around 10). Again, more data, less error and less variability. Also shows the formula standard error for mean approximately $s / \sqrt{n} = 66 / \sqrt{30} \approx 12$ (close to 10 but not perfect)

5. Answers will vary. One sampling distribution came out to 115.5 to 154.8. The distance between them is 39.3 (around 40). Dividing by 4 gives an approximate standard error of 10. Yes it agrees with Empirical rule that says in a bell shape curve the middle 95% will corresponds to about 2 standard deviations above and below the center.

Conf int act 3

1. Answers may vary. Many students are surprised to find their random sample proportion (percentage) is very different than the population value of 0.5. The random sample percentages can vary between 0.1 and 0.9 (see picture). For example, if a student got a sample percentage of 0.8, then their random sample mean is 0.3 to high.

2. Most of the sample means on the board (see picture) are vastly different than the population percentage of 0.5 (50%). This happens because there is a lot of variability in random samples. Even though random sampling is one of the best ways to collect data, this does not guarantee that the random sample value will be close to the population value. Random samples are always different and usually very different than the population value.

3. If all we knew is one random sample, it would be practically impossible to predict the population value with any accuracy. Most of the random samples means were very far off. If all we knew is the random sample mean, it would be almost impossible to know the population value with any certainty.

4. The shape of sampling distribution is relatively bell shaped (nearly normal). Yes. The center of the sampling distribution is much closer to the population value of 0.5 and would be a much better approximation of the population value than one single random sample mean like 0.15

5. Answers will vary. If a class has 35 students and each calculates two sample percentages, then 95% of 70 = 0.95 x 70 = 66.5 (approx. 67).

So we want the middle 67 dots and must take off a total of 3 dots (70-67). So taking 1 and $\frac{1}{2}$ from each end gives us the approximate values of 0.7 and 0.25. The distance between them is 0.45. The empirical rule states that the middle 95% of a bell shaped curve corresponds to 2 standard deviations above and 2 standard deviations below the center. So that is a total of 4 standard deviations. Take the distance of 0.45 and divide by 4. This gives an approximate standard deviation of 0.1125. The standard deviation of a sampling distribution is called the "standard error". So the standard error is approximately 0.1125. (Answers may vary.)
Conf int act 4

1. Sampling variability is on display. Notice that most of the random sample proportions (percentages) are vastly different than the population percentage of 0.5. A point estimate is when people take a random sample value and tell people it is the population value. This is horrible. Most of these random sample percentages are not even close to the correct population value.

2. Answers will vary. That is the point. Random samples are always different and always different than the population value. Students can take as many samples as they want. For example they may take 2000 random samples. As the number of samples in the distribution increases, it is becoming more and more bell shaped (central limit theorem in action).

3. Answers will vary. The center of the distribution gets more and more accurate as the number of random samples increase. This is a general principle in stats. The more random data you have, the less the error. The center of the distribution for example might be 0.502. This is extremely close to the population. While if we look at 1 random sample, it may be dramatically off. (One random sample percentage was 0.25) So to get a really good estimate of the population percentage all you need is thousands of random samples. (This is usually not possible. In real life, you may have only 1 random sample.)

4. The standard deviation of a sampling distribution is called the standard error. Answers will vary but most standard errors will be around 0.10 (One example was 0.110)

Note the difference between the standard deviation (of 1 random data set) and the standard error (standard deviation of thousands of sample values). Again, more data, less error and less variability.

5. Random sample answers will always vary. (That is the point.) One sampling distribution gave the numbers 0.300 and 0.700. The distance between them is 0.4. Dividing by 4 gives an approximate standard error of 0.10. Yes it agrees with Empirical rule that says in a bell shape curve the middle 95% will corresponds to about 2 standard deviations above and below the center.

Conf int act 5

1a
\[ \hat{p} = 39\% \]
MOE = 2.5%
Interval notation: (36.5%, 41.5%) OR (0.365, 0.415)
Inequality notation: 36.5% < \( p \) < 41.5% OR 0.365 < \( p \) < 0.415

1b
\[ \bar{x} = 69.2 \text{ in} \]
MOE = 1.9 in
Interval notation: (67.3 inches, 71.1 inches)
Inequality notation: 67.3 < \( \mu \) < 71.1

1c
\[ \hat{p} = 4.7\% \]
MOE = 1.2%
Interval notation: (3.5%, 5.9%) OR (0.035, 0.059)
Inequality notation: 3.5% < \( p \) < 5.9% OR 0.035 < \( p \) < 0.059
2a: Conf Int (0.731, 0.764)

\[ \hat{p} = \text{avg of } 0.731 \text{ & } 0.764 = 0.7475 \]

\[ \text{MOE} = 0.7475 - 0.731 = 0.0165 \]

95% confidence level: 95% of the confidence intervals contain the population proportion of cows having the disease, and 5% of the confidence intervals don’t contain the population proportion.

95% confidence interval: We are 95% confident that the population proportion of cows having the disease is between 73.1% and 76.4%

2b: Conf Int (13.4, 17.2)

\[ \bar{x} = \text{avg of } 13.4 \text{ & } 17.2 = 15.3 \]

\[ \text{MOE} = \frac{17.2 - 13.4}{2} = 1.9 \]

99% confidence level: 99% of the confidence intervals contain the population mean miles driven, and 1% of the confidence intervals don’t contain the population mean.

99% confidence interval: We are 99% confident that the population mean miles driven is between 13.4 miles & 17.2 miles.

2c: Conf Int (0.068, 0.083)

\[ \hat{p} = \text{avg of } .068 \text{ & } .083 = 0.0755 \text{ or } 7.55\% \]

\[ \text{MOE} = 7.55\% - 6.8\% = .75\% \]

90% confidence level: 90% of the confidence intervals contain the population mean miles driven, and 10% of the confidence intervals don’t contain the population proportion.

90% confidence interval: We are 90% confident that the population proportion of people who will vote for the Independent party candidate is between 6.8% & 8.3%.

2d: Conf Int (8.35, 10.21)

\[ \bar{x} = \text{avg of } 8.35 \text{ & } 10.21 = 9.28 \text{ gal} \]

\[ \text{MOE} = 10.21 - 9.28 = 0.93 \text{ gal} \]

95% confidence level: 95% of the confidence intervals contain the population mean miles driven, and 5% of the confidence intervals don’t contain the population mean.

95% confidence interval: We are 95% confident that the population mean amount of milk is between 8.35 gallons and 10.21 gallons.
Conf int act 6

1. Use StatCrunch to find the two Z-scores that corresponds to the middle 95%. Draw a picture showing the Z-scores and 95%. Remember that the area under the curve between these Z-scores must be 0.95.

*Do you remember what mean and standard deviation we use to find Z-scores on StatCrunch? Mean = 0 & Standard Deviation = 1*
2. If we use a 99% confidence level instead of 95% do you think the Z-score will be more or less than + or - 2?

Draw a picture and explain why you think so. Now use StatCrunch to find the two Z-scores that we could use to calculate a 99% confidence interval.

*How well did your first guess agree with what we found on StatCrunch?*
3. Now repeat #2, but use a 90% confidence interval.

*If we use a 90% confidence level instead of 95% do you think the Z-score will be more or less than \(+\) or \(-\) 2?*

Draw a picture and explain why you think so. Now use Statcrunch to find the two Z-scores that we could use to calculate a 90% confidence interval.

*How well did your first guess agree with what we found on Statcrunch?*

\[ Z = \pm 1.645 \]

Let’s Summarize the Z-score critical values that we found. These are important to memorize as we will be using them constantly in inferential statistics.

<table>
<thead>
<tr>
<th>Confidence Level</th>
<th>Z score for the confidence interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>90%</td>
<td>( Z_c = \pm 1.645 )</td>
</tr>
<tr>
<td>95%</td>
<td>( Z_c = \pm 1.96 )</td>
</tr>
<tr>
<td>99%</td>
<td>( Z_c = \pm 2.576 )</td>
</tr>
</tbody>
</table>
1. A random sample of 650 high school students has a normal distribution. The sample mean average ACT exam score was 21 with a 3.2 sample standard deviation. Construct a 99% confidence interval estimate of the population mean average ACT exam.

\[
\bar{x} \pm Z_{c} \frac{s}{\sqrt{n}} = 21 \pm (2.576) \left( \frac{3.2}{\sqrt{650}} \right) \approx 21 \pm 0.32
\]

which means

\[
21 - 0.32 < \mu < 21 + 0.32
\]

20.68 < \mu < 21.32

2. A random sample of 200 adults found that they had a sample mean temperature of 98.2 degrees and a standard deviation of 0.18 degrees. Construct a 95% confidence interval estimate of the population mean body temperature of adults. Does the confidence interval indicate that normal body temperature could be 98.6 degrees? No, since the confidence interval doesn’t contain 98.6°F.

\[
\bar{x} \pm Z_{c} \frac{s}{\sqrt{n}} = 98.2 \pm (1.96) \left( \frac{1.8}{\sqrt{200}} \right) \approx 98.2 \pm 0.249
\]

which means

\[
98.2 - 0.249 < \mu < 98.2 + 0.249
\]

97.95°F < \mu < 98.45°F

3. A random sample of 315 adults found that the sample mean amount or credit card debt was $435 with a standard deviation of $106. Construct a 90% confidence interval estimate of the population mean amount of credit card debt.

\[
\bar{x} \pm Z_{c} \frac{s}{\sqrt{n}} = 435 \pm (1.645) \left( \frac{106}{\sqrt{315}} \right) \approx 435 \pm 9.82
\]

which means

\[
435 - 9.82 < \mu < 435 + 9.82
\]

$425.18 < \mu < 444.82$
4. In a random sample of 72 adults in Santa Clarita, CA, each person was asked if they support the death penalty. 31 adults in the sample said that they do support the death penalty. What was the sample proportion of adults in Santa Clarita that support the death penalty? \(0.43\)

Now calculate a 95% confidence interval population estimate of people in Santa Clarita that support the death penalty. Remember to use the appropriate critical value Z-score for each.

\[
\hat{p} \pm Z_c \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 0.43 \pm 1.96 \sqrt{\frac{0.43(1-0.43)}{72}} = 0.43 \pm 0.114
\]

which means

\[
0.43 - 0.114 < p < 0.43 + 0.114
\]

\[
0.316 < p < 0.544 \quad OR \quad 31.6\% < p < 54.4\%
\]

5. In a random sample of 400 Americans, each person was asked if they are satisfied with the amount of vacation time they given by their employers. 84% of them said that they were not satisfied with their vacation time. Calculate the following. What was the sample proportion of Americans that were not satisfied with their vacation time? \(0.84\)

Now construct a 99% confidence interval in order to estimate the percent of Americans that are not satisfied with their vacation time.

\[
\hat{p} \pm Z_c \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 0.84 \pm 2.576 \sqrt{\frac{0.84(1-0.84)}{400}} = 0.84 \pm 0.047
\]

which means

\[
0.84 - 0.047 < p < 0.84 + 0.047
\]

\[
0.793 < p < 0.887 \quad OR \quad 79.3\% < p < 88.7\%
\]
6. What percent of eligible Americans vote? In 2008, a random sample of 3000 American adults that were eligible to vote was taken and we found that 2040 of them voted. Construct a 90% confidence interval estimate of the population percent of Americans that vote.

\[
\hat{p} \pm Z_c \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}} = 0.68 \pm 1.645 \sqrt{\frac{0.68(1 - 0.68)}{3000}} = 0.68 \pm 0.014
\]

which means

\[
0.68 - 0.014 < p < 0.68 + 0.014 \quad \text{OR} \quad 66.6\% < p < 69.4\%
\]

Now construct another confidence interval. This time construct a 90% confidence interval estimate of the population percent of Americans that do not vote. Hint: For the “do not vote” group, the sample percent will change.

\[
\hat{p} \pm Z_c \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}} = 0.32 \pm 1.645 \sqrt{\frac{0.32(1 - 0.32)}{3000}} = 0.32 \pm 0.014
\]

which means

\[
0.32 - 0.014 < p < 0.32 + 0.014 \quad \text{OR} \quad 30.6\% < p < 33.4\%
\]

**Conf int act 8**

#1

90% Confidence Interval: \(20.79 < \mu < 21.21\)

Interpretation:

We are 90% confident that the population mean average of ACT exam scores is between 20.79 and 21.21.

95% Confidence Interval: \(20.75 < \mu < 21.25\)

Interpretation:

We are 95% confident that the population mean average of ACT exam scores is between 20.75 and 21.25.
95% Confidence Interval: $97.95^\circ F < \mu < 98.45^\circ F$
Interval Interpretation: We are 95% confident that the population mean body temperature of adults is between $97.95^\circ F$ and $98.45^\circ F$.

99% Confidence Interval: $97.87^\circ F < \mu < 98.53^\circ F$
Interval Interpretation: We are 99% confident that the population mean body temperature of adults is between $97.87^\circ F$ and $98.53^\circ F$.

Do the confidence intervals indicate that normal body temperature could be $98.6^\circ F$? No, since the confidence intervals do not contain $98.6^\circ F$.

90% Confidence Interval: $425.18 < \mu < 444.82$
Interval Interpretation: We are 90% confident that the population mean amount of credit card debt is between $425.18 and $444.82.

95% Confidence Interval: $423.29 < \mu < 446.71$
Interval Interpretation: We are 95% confident that the population mean amount of credit card debt is between $423.29 and $446.71.

Mean = 2.87 min  Standard deviation = 1.38 min  Shape: Symmetric
95% Confidence Interval: $2.27 \text{ min} < \mu < 3.47 \text{ min}$
Interval Interpretation: We are 95% confident that the population mean number of minutes Starbucks customers have to wait for their drink is between 2.27 minutes and 3.47 minutes.

90% Confidence Interval: $241.76\text{ ft} < \mu < 254.24\text{ ft}$
Interval Interpretation: We are 90% confident that the population mean height of Redwood trees is between 241.76 feet and 254.24 feet.

90% confidence:
90% of the confidence intervals contain the population mean height of Redwood trees, and 5% of the confidence intervals don’t contain the population mean.

95% Confidence Interval: $27.74 \text{ yrs} < \mu < 30.27 \text{ yrs}$
Interval Interpretation: We are 95% confident that the population mean age of UCLA students is between 27.74 years and 30.27 years.
#7
Mean = $3.93  
Standard deviation = $1.13  
Shape: Slightly right skewed
99% Confidence Interval: $3.40 < \mu < $4.46
Interval Interpretation:
We are 99% confident that the population mean price of a hamburger is between $3.40 and $4.46.

99% confidence: 
99% of the confidence intervals contain the population mean price of a hamburger, and 1% of the confidence intervals don’t contain the population mean.

#8
90% Confidence Interval: 0.666 < p < 0.694  
OR  
66.6% < p < 69.4%
Interpretation:
We are 90% confident that the population percent of Americans that voted is between 66.6% and 69.4%.

99% Confidence Interval: 0.658 < p < 0.702  
OR  
65.8% < p < 70.2%
Interpretation:
We are 99% confident that the population percent of Americans that voted is between 65.8% and 70.2%.

#9
What was the sample proportion of adults in Santa Clarita that support the death penalty? \( \frac{31}{72} \approx 0.43 \) or 43%  
What is the standard error for this sampling distribution? 0.058
90% Confidence Interval: 0.335 < p < 0.525  
OR  
33.5% < p < 52.5%
Interpretation:
We are 90% confident that the population proportion of people in Santa Clarita that support the death penalty is between 33.5% and 52.5%.

95% Confidence Interval: 0.316 < p < 0.545  
OR  
31.6% < p < 54.5%
Interpretation:
We are 95% confident that the population estimate of people in Santa Clarita that support the death penalty is between 31.6% and 54.5%.

#10
What was the sample proportion that was satisfied with their vacation time? 16% or 0.16  
What was the standard error for the sampling distribution? 0.018
99% Confidence Interval (Satisfied): 0.113 < p < 0.207  
OR  
11.3% < p < 20.7%
Interpretation:
We are 99% confident that the population estimate of Americans that are satisfied with their vacation time is between 11.3% and 20.7%.

99% Confidence Interval (NOT Satisfied): 0.793 < p < 0.887  
OR  
79.3% < p < 88.7%
Interpretation:
We are 99% confident that the population estimate of Americans that are not satisfied with their vacation time is between 79.3% and 88.7%.
#11
a) As the confidence level gets higher, does the confidence interval get narrower or wider? Why? 
   As per below diagram, if the confidence level gets higher, then the confidence interval gets wider.

b) As the confidence level decreases, what happens to the margin of error? Why? 
   As per shown diagram, if the confidence level decreases, then the margin of error decreases.

c) As the sample size increases, what happens to the standard error? Why? 
   As the sample size increases, the standard error decreases. 
   $n$ (sample size) is in the denominator of the standard error formula. Thus, as the denominator gets larger, 
   SE gets smaller.

Conf Int Act 9

1. Each random sample mean was very different and all were different than the population mean. This is sampling 
   variability at work. In fact, if the sample mean was so far off from the population mean, the confidence interval 
   may not contain the population mean.

2. No. Not all confidence intervals contained the population values. Capturing or containing the population value 
   means that the actual population value is in between the two numbers in the confidence interval. The green 
   confidence intervals did contain the population value, but the red confidence intervals failed to contain the 
   population value.

3. Answers will vary. A student that took 200 intervals may find that anywhere from 187 to 193 of them did 
   contain the population value.

4. Answers will vary. A student that took 200 intervals may find that about 8, 9, 10 or 11 of them did not contain 
   the population value.

5. As the number of samples increased, the percentage of intervals that contain the population value seemed to 
   get closer to 95%.

6. Answers may vary. Students should understand that the word “confidence” in many ways is an admission that 
   the population value may not be in between the two numbers in the confidence interval. About 95% of confidence 
   intervals contain the population value (green) and about 5% of them did not (red)
7. Each random sample percentage was very different and all were different than the population percentage. This is sampling variability at work. In fact, if the sample percentage was so far off from the population percentage, the confidence interval may not contain the population percentage.

8. No. Not all confidence intervals contained the population values. Capturing or containing the population value means that the actual population value is in between the two numbers in the confidence interval. The green confidence intervals did contain the population value, but the red confidence intervals failed to contain the population value.

9. Answers will vary. A student that took 200 intervals may find that anywhere from 187 to 193 of them did contain the population value.

10. Answers will vary. A student that took 200 intervals may find that about 8, 9, 10 or 11 of them did not contain the population value.

11. As the number of samples increased, the percentage of intervals that contain the population value seemed to get closer to 95%.

12. Answers may vary. Students should understand that the word “confidence” in many ways is an admission that the population value may not be in between the two numbers in the confidence interval. About 95% of confidence intervals contain the population value (green) and about 5% of them did not (red).

Conf int act 10

1. df = 5-1 = 4
   90%: T = 2.1319  (A lot larger than z-score of 1.645)
   95%: T = 2.7764  (A lot larger than z-score of 1.96)
   99%: T = 4.6041  (A lot larger than z-score of 2.576)

2. df = 17-1 = 16
   90%: T = 1.7459  (Slightly larger than z-score of 1.645)
   95%: T = 2.1199  (Slightly larger than z-score of 1.96)
   99%: T = 2.9208  (Slightly larger than z-score of 2.576)

3. df = 45-1 = 44
   90%: T = 1.6802  (Almost the same as z-score of 1.645)
   95%: T = 2.0154  (Almost the same as z-score of 1.96)
   99%: T = 2.6923  (Almost the same as z-score of 2.576)

4. As the sample size and degree of freedom increase, the t-scores get closer and closer to the famous critical value z-scores. They are really different for small sample sizes.
1. This is voluntary response data and is filled with bias. We do not know the shape of this data, but the sample size was large enough. However it was not random data and was collected incorrectly. This data will not represent the population, so making a confidence interval would be a waste of time.

2. The data was randomly selected. The sample size was very small and may have some sampling bias. However since the data was bell shaped, it does meet the 30 or normal minimum requirement to make the confidence interval. This does not mean we are very happy with the small sample size. A 95% confidence interval estimate was ($86693, $92607). So we are 95% confident that the average price of a home in Oklahoma City is between $86693 and $92607.

3. The shape is skewed right and not normal. However the sample size was 45 and is greater than 30. So the data does meet the 30 or normal requirement. However he did not pick stores randomly. This is convenience data and will not represent the population, so making a confidence interval would be a waste of time.

4. Rachael collected some good data. She attempted to do a census which is better than random. The sample size is 213 which is over 30. We do not know shape, but even if the data was skewed, it would still pass the 30 or normal requirement. StatCrunch gave the following 95% confidence interval: ($1300, $1350). So we are 95% confident that the average price of a 1 bedroom apartment in Northridge is in between $1300 and $1350.

5. Mike used a random cluster technique, which was acceptable. The data set was skewed right and so was not bell shaped. However the sample size of 63 is over 30, so the data does pass the minimum requirement of 30 or normal. StatCrunch gave the following 95% confidence interval: (1.149 thousand, 1.451 thousand) So we are 95% confident that people in Los Angeles would be willing to pay between $1,149 and $1,451 for an NFL football team.

6. Marsha did have a random sample. The number of successes was 37 and the number of failures was 123-37 = 86. So there are at least 10 successes and at least 10 failures. Also people randomly called are not likely to know each other or be related in any way. StatCrunch gave the following 95% confidence interval: (0.220, 0.382) We are 95% confident that between 22.0% and 38.2% of voters will vote for the republican candidate in the next election.

7. This data is full of bias. First of all the stores and people were not randomly selected. Also though we have 71 failures, there was only 8 successes. So it fails the at least 10 success and failures criteria. Also people in stores at the same time may know each other and influence their answer about using a pipe. This data will not represent the population and so would be a waste of time to make a confidence interval.

8. This data is also biased. Though the data set is large enough (112 successes and 247-112 = 135 failures), the data is voluntary response. The students were not randomly selected. This data will not represent the population and so would be a waste of time to make a confidence interval.

9. This data is full of bias. First of all, the students were not randomly selected. This was convenience data and is very biased. Also though we have 26 successes, there was only 8 failures. So it fails the at least 10 success and failures criteria. Also people in same class in the same time may know each other and influence their answer about going to COC. There may also be some response bias, as students may not want to answer truthfully about their plans after high school. This data will not represent the population and so would be a waste of time to make a confidence interval.
Conf Int Act 12

1. The “distribution of sample means” is the same as a sampling distribution. A sampling distribution is when we look at lots and lots of sample values to try to understand how much variability random samples have from the population. In this case the sample values were sample means.

2. Yes the central limit theorem indicates that if the original population is bell shaped (normal) then the sampling distribution for means or percentages will also be bell shaped.

3. If the population is skewed, then the sampling distribution will only be bell shaped if the sample size is sufficiently large. For means the minimum requirement is at least 30 and for percentages, we need at least 10 successes and at least 10 failures. We get our assumptions from the central limit theorem.

4. Central Limit Theorem: The distribution of sample means (or sample percentages) will be normally distributed for sufficiently large data sets.

5. There are many consequences of the central limit theorem. We are allowed to use bell shaped curves to estimate probabilities (P-value) as long as the population is normal or the data set is sufficiently large. This is where we get the assumptions for using sample means to estimate the population with confidence intervals (random, 30 or normal). We also get the assumptions for using sample percentages to estimate the population percentages with confidence intervals (random, at least 10 success, at least 10 failures).

Conf Int Act 13

1. The data was randomly selected from the population of coffee data. Students checked the shape of the samples with histograms and found that many were skewed right. However since the data sets have a sample size of 30, it does pass the bare minimum requirements. Picking a random sample of Brazilian coffee and Columbian coffee would be independent. The coffee prices in each row do not come from the same month. If we randomly picked months and got both the Columbian and Brazilian prices in that month it would be matched pairs.

2. Answers will vary from student to student. For example, one student got a sample mean for Columbian coffee at 119 cents and a sample mean for Brazilian at 100 cents. So subtracting Columbian – Brazilian gives +19 cents. This was not too far from 22.721 cent population difference. However other students got a sample difference of -6 cents. This was very different. Again, this is sampling variability. Random samples will be different from each other and may be very different than the population.

3. A positive difference tells us that the value from the first sample was greater than the value from the second sample. A negative difference tells us that the value from the first sample was less than the value from the second sample. A difference of zero would indicate that the two sample values were the same.

4. (See picture) There was a lot of sampling variability in the differences. Some students got positive differences indicating that Columbian coffee might be more expensive. Some students got negative differences indicating that Columbian coffee might be less expensive and Brazilian more expensive. If we only knew one sample difference it would be almost impossible to know the population difference with any accuracy.

5. 95% of 60 = 0.95 x 60 = 57. So we would want the middle 57 dots and would take off 1 and ½ dots from each side. Answers will vary. Usually the middle 57 dots are between -11 cents and 45.5 cents.

6. -11 and 45.5 are about 56.5 apart. Dividing by 4 (Empirical rule) gives us an approximate standard error of about 14. The data was not very bell shaped though. The sampling distribution looks slightly skewed left.
7. (Each student had different sample data so answers will vary.) The data was randomly selected from the population of coffee data. Students checked the shape of the samples with histograms and found that many were skewed right. However since the data sets have a sample size of 30, it does pass the bare minimum requirements. Picking a random sample of Brazilian coffee and Columbian coffee would be independent. The coffee prices in each row do not come from the same month.

8. Flipping coins 20 times with the right hand and 20 times with the left hand.

9. Answers will vary. If a student got 0.4 with the right hand and 0.6 with the left than the difference would be -0.2. This difference would be 0.2 below zero.

10. Remember a positive difference means group 1 (right hand) had a higher percentage. A negative difference means that group 1 had a lower percentage.

11. (See picture) There was a lot of sampling variability in the differences. Some students got positive differences indicating that the right hand had a higher percentage of tails. Some students got negative differences indicating that right hand had a lower percentage. If we only knew one sample difference like 0.3, it would be almost impossible to know the population difference with any accuracy.

12. Answers will vary. For example if there are 35 students in the class, and each did the coin flipping once with each hand then 95% of 35 would be 0.95 x 35 = 33.25 so we would take 1 dot off of each end to get the middle 33. Estimates will vary but an example might be between -0.3 and +0.35

13. -0.3 and +0.35 are 0.65 apart. Dividing by 4 (Empirical rule) gives an approximate standard error of 0.16

14. Flipping coins were relatively random, but it did not meet the sample size requirement. Most students did not get at least 10 successes and 10 failures when flipping 20 times. The sample sizes were not large enough to estimate the population.

Conf int act 14

1) The problem meets the assumptions for a population mean difference. Both data sets were random and even though the sample sizes were below 30, they were bell shaped. The data was the same student measured twice so was matched pair data. A 90% confidence interval estimate of the difference between the after – before was ( +4.4 , +7.2 ) Because it was all positive this indicates that the after scores were significantly higher than the before scores. So there was a significant difference. In fact, we are 90% confident that students after taking the prep class score between 4.4 and 7.2 points higher on the ACT exam than before taking the prep class. The prep class was effective in raising ACT scores.

2. The problem meets the assumptions for population percent difference. Both data sets were random with at least 10 success and at least 10 failures. We let men be population 1 and women be population 2. The 90% confidence interval for the difference between the percentages was (-0.033 , +0.028). The 90% confidence interval for the difference between the percentages was (-0.039 , +0.034). Both indicate the same thing. Because the interval includes negative and positive numbers and zero, we are 95% or 90% confident that there is no significant difference between the percent of men with at least one tattoo and the percent of women with at least one tattoo.
3. The problem meets the assumptions for a population mean difference. Both data sets were over 30 and random (did not need to check shape since the data sets were over 30). These were independent groups. If we let people that do not live with smokers be population 1 and people that live with smokers be population 2, then a 95% confidence interval for the difference was (-23.98 ng/mL, -18.62). Because the interval is all negative, we know that people that do not live with smokers have a significantly lower cotinine level than the people that live with smokers. The sentence for the interval would be: we are 95% confident that people that do not live with smokers will have a cotinine level between 18.62 ng/mL and 23.98 ng/mL lower than those that live with smokers.

4. The problem meets the assumptions for population percent difference. Both data sets were random with at least 10 success and at least 10 failures. We let men be population 1 and women be population 2. The 95% confidence interval for the difference between the percentages was (0.047, 0.140). So we are 95% confident that the percent of men with a normal BMI is between 4.7% and 14% higher than the percent of women with a normal BMI. So there is a significant difference. The 99% confidence interval for the difference between the percentages was (0.032, 0.155). So we are 99% confident that the percent of men with a normal BMI is between 3.2% and 15.5% higher than the percent of women with a normal BMI. This shows there is a significant difference.

5. This data meets the assumptions for population mean difference. Both data sets were greater than 30 and random. Since it is the same 40 men measured twice, we know this is matched pair data. We let the systolic be population 1 and the diastolic be population 2. Going to statcrunch, t-stat, paired we get the 98% confidence interval was (+42.1, +49.3). So there is a significant difference. We are 98% confident that a man’s systolic blood pressure is between 42.1 and 49.3 points higher than a man’s diastolic blood pressure.

6. The problem meets the assumptions for population percent difference. Both data sets were random with at least 10 success and at least 10 failures. We let treatment group be population 1 (people that take the medicine) and placebo group be population 2 (people that do not take the medicine). The 97% confidence interval for the difference between the percentages was (-0.017, +0.075). So we are 97% confident that there is no significant difference between the treatment group and the placebo group. The 99% confidence interval for the difference between the percentages was (-0.026, + 0.084). So we are 99% confident that there is no significant difference between the treatment group and the placebo group. Both confidence intervals indicate that the medicine is not effective in lowering cholesterol since there was not a significant difference between people do and do not take the medicine.

7. The problem meets the assumptions for population mean difference. Both data sets are random and the sample size is greater than 30. A random sample of men and women have nothing in common so this is an independent case. Using Statcrunch, T-stat, 2 sample gives the following 95% confidence interval of the difference. We made women population 1 and men population 2. (-263.4, -45.3) Since the interval is completely negative, indicates that population 1 (women) has a significantly lower cholesterol than population 2 men. So we are 95% confident that the average cholesterol for women is between 45.3 and 263.4 point lower than men.
8. The problem meets the assumptions for population percent difference. Both data sets were random with at least 10 success and at least 10 failures. We let the 2003 data be population 1 and the 2008 data be population 2. Calculating a 90% and 95% confidence interval difference yields (0.441, 0.484) and (0.436, 0.488). Both are positive telling us that in 2003 the percent of americans that supported the war in Iraq was much higher than the percent in 2008. In fact we are 90% confident that the percent of Americans that supported the war in Iraq in 2003 was between 44.1% and 48.4% higher than the percent in 2008. We are 95% confident that the percent of Americans that supported the war in Iraq in 2003 was between 43.6% and 48.8% higher than the percent in 2008.

9. If we decrease the confidence level, the interval gets smaller. It is conceivable that you could have a wide interval at 99% that includes negative and positive numbers and so has “no significant difference”. At 90% the interval could get smaller and now be completely positive meaning that we now do have a significant difference. If it is already not significant at 99% then it will still be not significant at 90% level. If we increase the confidence level, the interval gets larger. It is conceivable that you could have a narrow interval at 90% that is all positive numbers or all negative numbers and so has a “significant difference”. When we increase the confidence level to 99% the interval could get wider and now include both negative and positive numbers. Hence the 99% could be “no significant difference. If it is already significant at 90% then it will still be significant at 99% level.

Review Sheet Answers - Confidence Interval/Sampling Distribution

1. Sampling variability implies that when we take different random samples we get different means and percentages. They do not come out the same. Sampling variability also implies that a random sample mean or percent will not be the same as the population mean or population percent. In fact we can calculate the margin of error i.e. the difference between the sample value and the population value. Adding and subtracting the sample value and margin of error gives us confidence intervals. It is almost impossible to estimate a population value from a single sample. The news is often very inaccurate when they use a sample value and tell the public it is a population value.

2. Sampling Distribution: Take a lot of random samples and calculate the mean or percent from each sample. We then make a graph of all of the thousands of sample means or sample percentages. (We can analyze the shape, center and spread of the distribution to better understand the population.)

   Standard Error: Standard deviation of the sampling distribution. Not the standard deviation of a single data set, but the standard deviation of the sample values for thousands of data sets.

   Margin of Error: How far we think one sample value could be from the population value. Margin of error is calculated by multiplying a z-score or t-score times the standard error.

   95% confidence: 95% of confidence intervals created contain the population value and 5% of them don’t.

   Confidence Interval: two numbers that we think the population value is in between. “We are 95% confident that the population value is between ## and ##.”
3. A sampling distribution is when we take a lot of random samples and calculate the mean or percent from each sample. We then make a graph of all of the thousands of sample means or sample percentages. (We can analyze the shape, center and spread of the distribution to better understand the population.) Standard deviation is the variability in one data set. Standard Error is a measure of variability for the sampling distribution (thousands of data sets). Standard Error measures the variability in sample means or sample percentages. Calculate 95% of the dots in the sampling distribution, then estimate the values that the middle 95% of the dots fall in between. If we divide the 95% confidence interval values into 4 sections, you get an approximate standard error. And the center of the distribution is very close to the population value. Using technology, you can also have the computer find the standard deviation (standard error) of the sampling distribution directly which is more accurate.

4. CLT: Central Limit Theorem: If the samples are large enough, the distribution of sample means or sample percentages will be normal even if the population is skewed. If the original population is close to bell shaped (nearly normal), then any size sample will give means that will be also nearly normal. For a distribution of means to be bell shaped we like samples to be at least 30 or nearly normal. For a distribution of percentages we like the sample to have at least 10 success and at least 10 failures. The larger the data set the smaller the standard error. Standard deviation of 1 data set is quite a bit larger than the standard error (stand dev from sampling distribution).

5. a) Sample mean = 6.8 Lbs, Margin of Error = 1.7 Lbs
   Confidence Interval (5.1 Lbs, 8.5 Lbs) WE are 95% confident that the population mean average weight of this breed of small dog is in between 5.1 pounds and 8.5 pounds.

b) Sample percent = 11.2% (or 0.112),
   Margin of Error = 1.3% (or 0.013)
   Confidence Interval: (9.9%, 12.5%) or (0.099, 0.125)
   We are 95% confident that between 9.9% and 12.5% of seniors worldwide have Alzheimer’s disease.

c) Sample mean difference = -5.3 inches
   Margin of Error = 1.7 inches
   Confidence interval: (-7.0, -3.6)
   We are 95% confident that the average height of women is between 3.6 and 7 inches less than the average height of men.

d) Sample Percent difference = 48.9% or 0.489
   Margin of Error = 2.6% or 0.026
   Confidence interval: (46.3%, 51.5%) or (0.463, 0.515)
   We are 95% confidence that the percent of people with high blood pressure in Bulgaria is between 46.3% and 51.5% higher than the percent of people with high blood pressure in Finland.
6. a) We are 95% confident that the mean average price of gas in the U.S. in November 2015 is between $2.16 and $2.24.
   sample mean = \((2.24 + 2.16) / 2 = 2.20\)
   margin of error = \((2.24 - 2.16) / 2 = 0.08 / 2 = 0.04\) (4 cents)

   b) We are 95% confident that the population percent chance of getting Lyme disease if bitten by a tick is between 1.2% and 1.5%.
   sample percent = \((0.015 + 0.012)/2 = 0.0135\) or 1.35%
   margin of error = \((0.015 - 0.012)/2 = 0.0015\) or 0.15%

   c) We are 95% confident that the amount of hemoglobin per decaliter of blood for men is between 1.6 g/dL and 2.0 g/dL greater than women.
   sample mean difference = 1.8 g/dL
   margin of Error = 0.2 g/dL

   d) We are 95% confident that world speed record times for men are between 9.65% and 10.35% lower than women’s speed record times.
   Sample percent difference = -10% (or -0.1)
   margin of error = 0.35% (or 0.0035)

7. As the confidence level increases, the margin of error also increases (larger z score) and the interval gets wider. As the confidence level decreases, the margin of error also decreases (smaller z score) and the interval gets narrower.  
   As the sample size increases, the standard error and margin of error decreases, which gives a narrower interval. If the sample size decreases, you have more error so the standard error and the margin of error increase, which gives us a wider interval.

8. a) The data does meet assumptions to check a population mean. The data is random and even though it is skewed the sample size is greater than or equal to 30. Using Statcrunch T-stat, 1 sample, with summary, we obtained the 95% confidence interval ( $42972.16 , $46359.84 ) . So we are 95% confident that the average yearly salary for k-12 teachers in the U.S. is between $42,972.16 and $46,359.84 .

   b) The data set was large enough, however this is voluntary response data and never representative of the population. We would be wasting our time trying to use this data to estimate a population percent or calculating a confidence interval. It needs to be random.

   c) The data does meet assumptions for calculating a population percent. The data was random and it had at least 10 people that were satisfied and at least 10 people that were not satisfied. Using Statcrunch , proportion stat, 1 sample, with summary, we obtained the following 95% confidence interval (0.678 , 0.835 ). So we are 95% confident that the population percent of patients that are satisfied with their care is between 67.8% and 83.5%.

   d) The data set was large enough, however this is convenience data and never representative of the population. We would be wasting our time trying to use this data to estimate the difference between two population percentages or calculating a confidence interval for the difference. It needs to be random.
e) The data does meet assumptions to check a population mean. Both data sets are random. The groups are independent of each other and both sample sizes are greater than or equal to 30. Using women’s BMI as population 1 and men’s BMI as population 2, we used Statcrunch T-stat, 2 sample, with data, we obtained the 95% confidence interval (−2.48 BMI points, +1.96 BMI points). Note: If you had made men’s BMI population 1 then you would have gotten (−1.96, +2.48). Both answers tell us the same thing. So we are 95% confident that there is no significant difference between women’s BMI and men’s BMI.